A brief introduction to

“ Lévy Process ”

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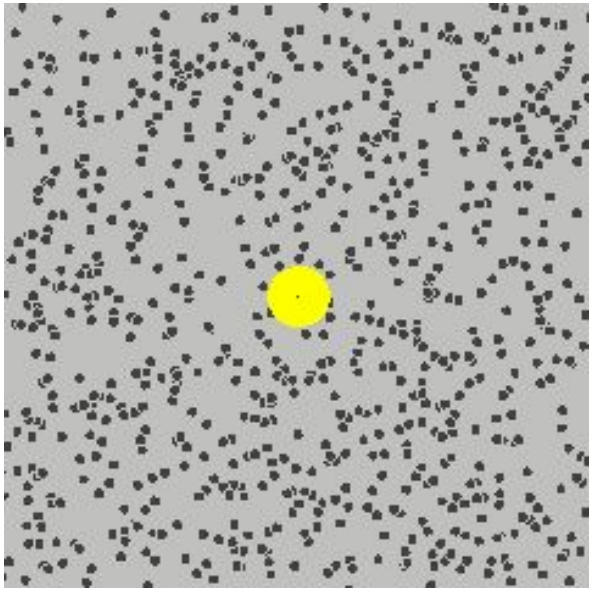
Random Walk or Brownian motion



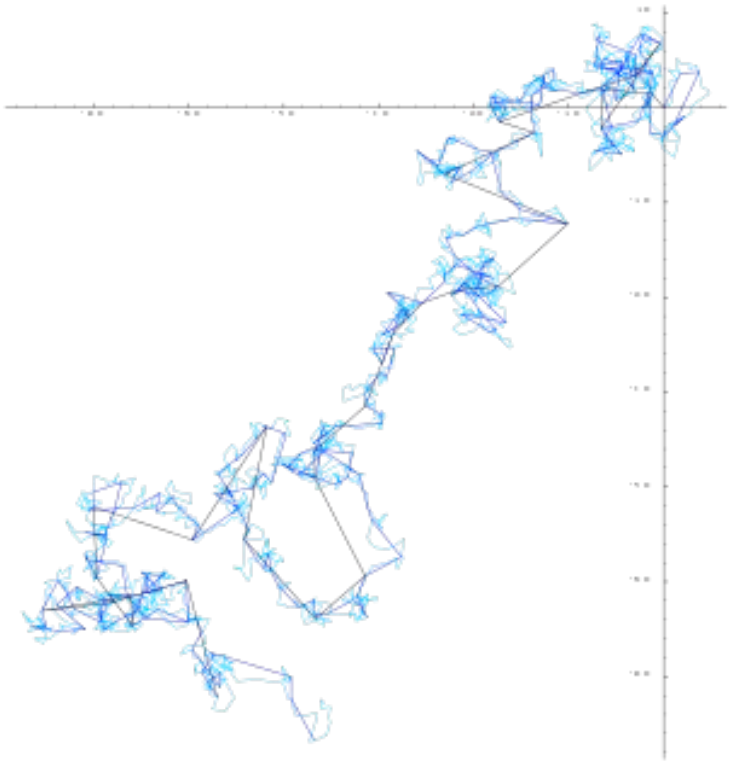
This is a simulation of the Brownian motion of a big particle (dust particle) that collides with a large set of smaller particles (molecules of a gas) which move with different velocities in different random directions.

wikipedia

Brownian motion

Three different views of Brownian motion, with 32 steps, 256 steps, and 2048 steps denoted by progressively lighter colors.

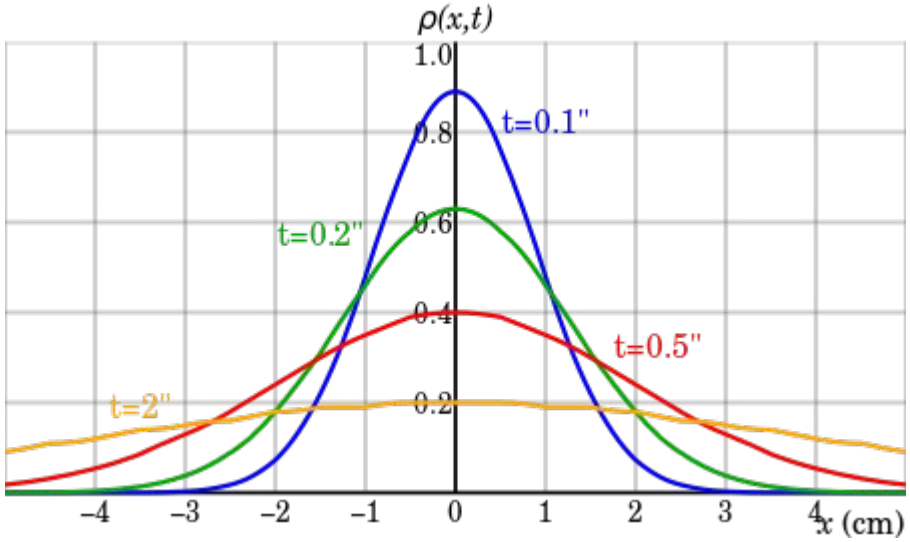
wikipedia



Brownian motion

1. Dirac delta function
2. Flatter and flatter
3. Uniform

Fick's Law of Diffusion: $\overline{x^2} = 2\,D\,t.$



wikipedia

Generalized Random Walk

step lengths during the walk are described by a 'heavy-tailed' probability distribution

Generalized RW

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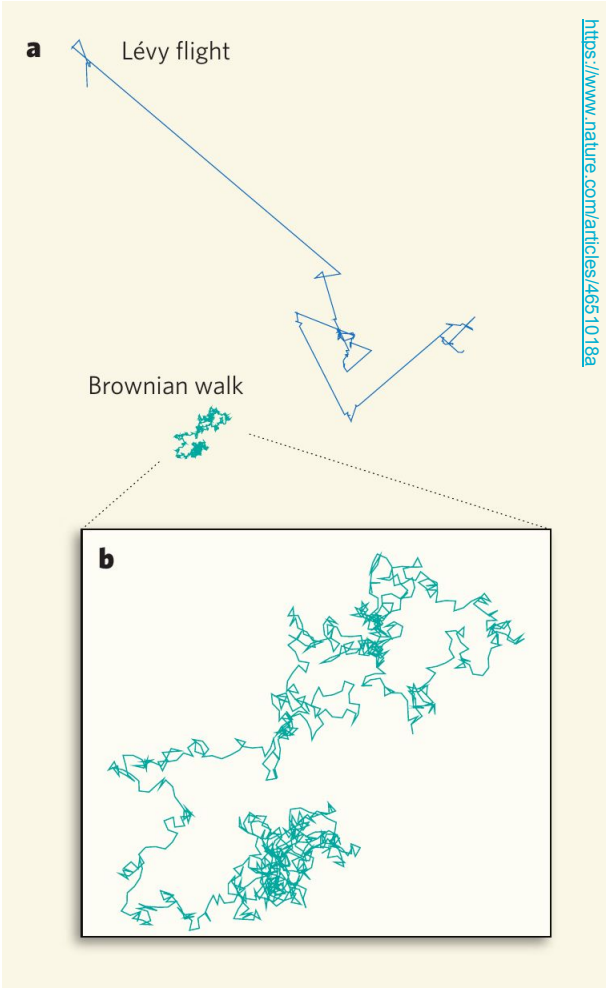
a, Normal diffusive random walk;
#Oversampling

b, Lévy random walk (Lévy flight).

$$P(l) \sim l^{-\mu}$$

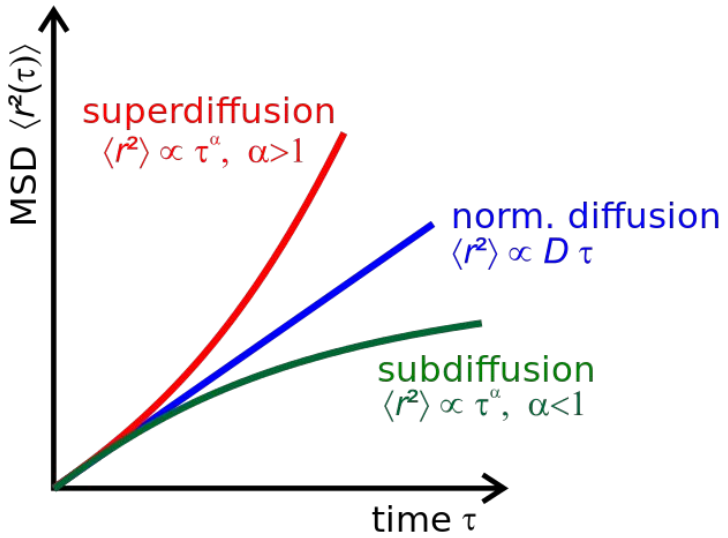
with $1<\mu\leq 3$.



non-Fickian or Anomalous Diffusion

Much of the interest in Lévy flights is due to their **superdiffusive** properties.

(MSD): σ_r^2:
amount of space the particle has "explored" in the system.



RW VS GRW

- **RW:**
  - Each step contributes equally to the average transport properties.

- **Lévy flight:**
  - Long steps are more frequent and make the dominant contribution to the transport.

They can describe all stochastic processes that are

scale invariant!

- Protein diffusion within cells
- Diffusion through porous media
- Animal foraging patterns[1]
- Distribution of human travel[2]
- Some aspects of earthquake behaviour[3]

Lévy flight foraging hypothesis

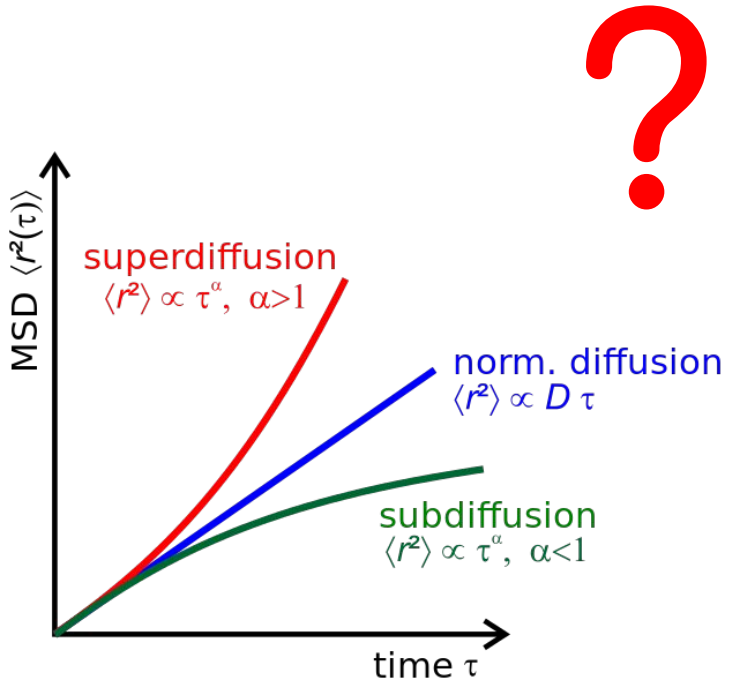
> *Since Lévy flights and walks can optimize search efficiencies, therefore* ***natural selection*** *should have led to adaptations for Lévy flight foraging.*

Lévy flight foraging hypothesis

1999: an **inverse square distribution of flight times or distances** could optimize the search efficiency under certain circumstances.[4]

- ★ Constant velocity search
- ★ Lévy flight path
- ★ Sparsely and randomly distributed revisitable targets
- ★ Absence of memory

- Did **humans disperse from Africa** superdiffusively rather than diffusively?
- Does **pollen from genetically modified crops** spread superdiffusively?
- What are the consequences if **influenza epidemics** spread superdiffusively?



References

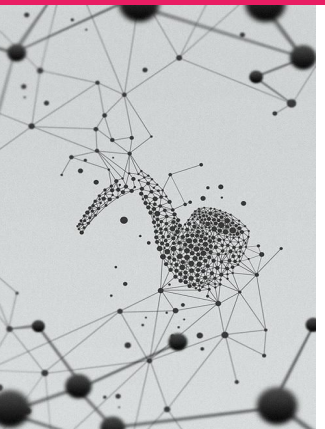
[1]: Bartumeus, F., Da Luz, M. G. E., Viswanathan, G. M. & Catalan, J. Animal search strategies: A quantitative random-walk analysis. *Ecology* **86**, 3078–3087 (2005)

[2]: Brockmann, D., Hufnagel, L. & Geisel, T. The scaling laws of human travel. *Nature* **439**, 462–465 (2006)

[3]: Corral, A. Universal earthquake-occurrence jumps, correlations with time, and anomalous diffusion. *Phys. Rev. Lett.* **97**, 178501 (2006)

[4]: Viswanathan, G. M.; Buldyrev, Sergey V.; Havlin, Shlomo; da Luz, M. G. E.; Raposo, E. P.; Stanley, H. Eugene (28 October 1999). "Optimizing the success of random searches". *Nature*. **401** (6756): 911–914.

Thank You :)



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